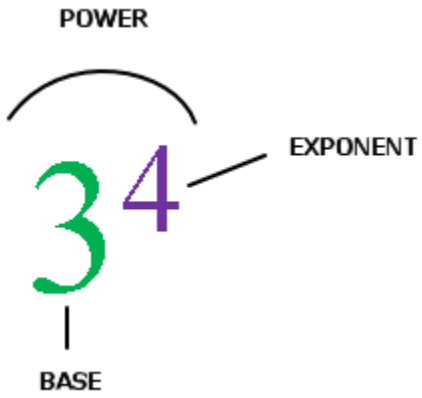
Rules of Exponents

In the expression, $\mathbf{x^n}$, $\boldsymbol{x}$ is referred to as the **base**, and $\boldsymbol{n}$ is called the **exponent**. The expression is read "**x to the nth power**."

An exponent tells you how many times to multiply the base times itself.



$$3 \cdot 3 \cdot 3 \cdot 3 = 81$$

Doing math with exponents looks scary at first, but with the help of a few rules, it's actually quite easy!

Here are 8 rules of exponents:

1. **Product Rule (same base):** When multiplying two exponents that have the same base, the base stays the same, and the exponents are added.

$$a^m * a^n = a^{m+n}$$

Example:

$$5^4 \cdot 5^2 = 5^{4+2} = 5^6 = 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 15{,}624$$

2. **Product Rule (different base):** When multiplying two exponents that have different bases, there is nothing that can be done to simplify the expression.

$$a^m * b^n = a^m * b^n$$

Example:

$$5^3 * 3^2 = 5 * 5 * 5 * 3 * 3 = 1125$$

3. Quotient Rule: When dividing exponents with the same base, subtract the exponents.

$$\frac{a^m}{a^n} = a^{m-n}$$

Example:

$$\frac{4^5}{4^3} = 4^{5-3} = 4^2 = 4 * 4 = 16$$

4. Negative Exponent Rule: Exponents are not always positive. When you see a negative exponent, you will divide 1 by that number.

$$a^{-m} = \frac{1}{a^m}$$ where $a \neq 0$

Example:

$$4^{-3} = \frac{1}{4^3} = \frac{1}{4 * 4 * 4} = \frac{1}{64}$$

5. Power Rule: When raising a power to a power, you multiply the exponents.

$$(a^m)^n = a^{m*n}$$

Example:

$$(4^2)^3 = 4^{2*3} = 4^6 = 4 * 4 * 4 * 4 * 4 * 4 = 4096$$

6. Expanded Power Rule: If an exponent is outside of parentheses, the exponent is applied to everything inside the parentheses. You must make sure to apply any other power rules in combination with the expanded power rule.

$$(ax)^m = a^m * x^m \text{ and } \left(\frac{ax}{by}\right)^m = \frac{a^m x^m}{b^m y^m}$$ where $b \neq 0$ and $y \neq 0$

Example:

$$\frac{(-3x^3)^3}{4} = \frac{-3^3 x^{3*3}}{4^3} = \frac{-3 * -3 * -3 * x * x * x * x * x * x * x * x * x}{4 * 4 * 4} = \frac{-27x^9}{64}$$

7. Zero Exponent Rule: Any base raised to the 0 power is equal to 1.

$$a^0 = 1$$

Example:

$$1{,}069^0 = 1$$

8. One Exponent Rule: Any base raised to the 1 power is equal to itself.

$$a^1 = a$$

Example:

$$9^1 = 9$$